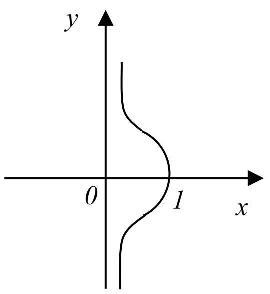
Question Number	Scheme	Marks
1.	Eliminating $\sqrt{\ }$: $\cos^2 x = 1 - \frac{1}{2}\sin 2x$	M1
	Using correct formulae to form equation in $\sin x$ and $\cos x$, or $\sec^2 x$ and $\tan x$, or $\sin 2x$ and $\cos 2x$	M1
	$\Rightarrow$ e.g. $\sin^2 x - \sin x \cos x = 0$ OR $1 = \sec^2 x - \tan x$	A1
	$\sin 2x \cos 2x = 0$ OR $\cos 2x + \sin 2x = 1$ (any form)	
	$\Rightarrow$ $\sin x(\sin x - \cos x) = 0$ OR $\tan x(\tan x - 1) = 0$ OR $R\sin(2x + \alpha) = 1$ or equiv	M1
	[M1 dependent on no wrong formulae being used]	
	$\Rightarrow$ $\sin x = 0$ and $\tan x = 1$; OR $\tan x = 0$ and $\tan x = 1$; or $\sqrt{2}\sin(2x + 45^\circ) = 1$	A1
	$\Rightarrow x = (0^\circ), 180^\circ$; $45^\circ, 225^\circ$	B1; A1
	Checking for spurious answers due to squaring	M1
	Only answers are 180° and 225°	A1
		(9 marks)

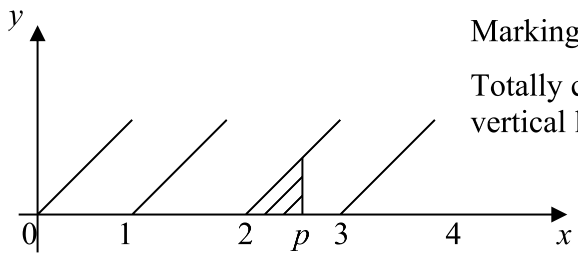
Question Number	Scheme	Marks
2. (*a*) (i)	$1 + 2x + 3x^2 + 4x^3 + \ldots$	B1
	$n + 1$	B1 (2)
(*b*)	$\sum_{n=1}^{\infty} nx^n = x + 2x^2 + 3x^3 + 4x^4 + \ldots\ldots\ldots\ldots$	
	$= x[1 + 2x + 3x^2 + 4x^3 + \ldots + \{(n+1)x^n\} + \ldots]$	M1
	$= \frac{x}{(1-x)^2}$ AG	A1 (2) cso
Alt.	$x(1-x)^{-2} = x + 2x^2 + 3x^3 + \{(n+1)x^{n+1}\} + \ldots = \sum_{n=1}^{\infty} n\,x^n$	M1A1
(*c*)	$\sum_{n=1}^{\infty}(an+1)x^n = x + x^2 + x^3 + x^4 + \ldots.. +$ $a\sum_{n=1}^{\infty} nx^n$ $\left[\sum_{1}^{\infty} x^n + \sum_{1}^{\infty} anx^n\right]$	M1
	$= \frac{x}{1-x} + \frac{ax}{(1-x)^2}$	A1
	$= \frac{x(1-x) + ax}{(1-x)^2}$; $= \frac{(a+1)x - x^2}{(1-x)^2}$ AG	M1;A1 (4) (cso)
Alt.	$\frac{(a+1)x - x^2}{(1-x)^2} = \{(a+1)x - x^2\}(1 + 2x + 3x^2 + \ldots.$	M1
	$= (a+1)x - x^2 + (a+1)2x^2 - 2x^3 \ldots..$	A1
	$= (a+1)x + (2a+1)x^2 + (3a+1)x^3 \ldots\ldots.$	M1
	$= \sum_{n=1}^{\infty}(an+1)x^n$	A1
(*d*)	Substituting a = 5, $x = \frac{1}{8}$, to give $\sum_{1}^{\infty}\frac{5n+1}{2^{3n}} = \frac{47}{49}$	M1A1 (2)
		(10 marks)

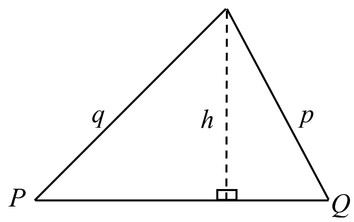
Question Number	Scheme	Marks
3. (*a*)	$f(2) = 8 - (k+4)(2) + 2k = 0 \Rightarrow$ curve passes through (2, 0)	B1 (1)
(*b*)	$f(x) = (x-2)(x^2 + 2x - k)$ {can gain in (a)}	M1
	Either $x^2 + 2x - k = 0$ has equal roots $\Rightarrow 4 = -4k \Rightarrow k = -1$	M1 A1
	(Roots –1, –1, 2) or perfect square	
	Or $x = 2$ is a solution of $x^2 + 2x - k = 0 \Rightarrow k = 8$	M1 A1 (5)
	(Roots 2, 2, –4)	
	[Marks as $(x-2)^2(x+a) \Rightarrow a = 4 \Rightarrow k = 8$ $(x-2)(x+b)^2 \Rightarrow b = 1 \Rightarrow k = -1$]	
	Differentiation approach: $\frac{dy}{dx} = 0 \Rightarrow x = (\pm)\sqrt{\frac{k+4}{3}}$	
	and use to attempt to find k	M1
	Set $\sqrt{\frac{k+4}{3}} = 2$ gives $k = 8$	M1 A1
	Set $f\left(-\sqrt{\frac{k+4}{3}}\right) = 0$ gives $k = -1$	M1 A1
	"*Roots*" *approach*: Roots are $\alpha, \alpha, -2\alpha$	M1
	Use relationships to find $\alpha = -1, \alpha = 2$	M1 A1
	Finding $k = -1$, $k = 8$	M1 A1
(c)	Relating to (b) to give $k = 8$ (or +ve numeric k, if one +ve, one –ve)	M1
	Attempting to find max (and min.) up to $x = \ldots$ (allow if still in k)	M1*
	[$\frac{dy}{dx} = 3x^2 - 12 = 0$, $x = -2$ for max.]	
	Max is $f(-2) = 32$	A1
	$\text{Min}_c < p < \text{max}_c$ (allow $\leq$ for M, give in diag., allow if still in k)	M1 dep*
	$0 < p < 32$ [A1√ requires 0 and candidates f(–2) but must be > 0]	A1√ (5)
		(11 marks)

Question Number	Scheme	Marks
4. (*a*)	Centre (r, 4), $\tan\theta = \frac{3}{4}$ or $OA = 4$ seen or implied anywhere	B1 B1
(i)	Method α Finding cords of A	
	[$A = (4k, 3k) \Rightarrow OA = 5k = 4$ or $(4\cos\theta, 4\sin\theta)$ with attempt at θ]	M1
	$A = (\frac{16}{5}, \frac{12}{5})$ can be written down	A1
	Complete method for r: $\frac{4 - y_A}{r - x_A} = -\frac{4}{3}$ or $(r - x_A)^2 + (4 - y_A)^2 = r^2$	M1
	$r = 2$	A1
Alt: Method β	$\updownarrow CN = 4\sin\theta + r\cos\theta$	M1
	Complete method to find r	
	$CN = 4$; with θ substituted	M1 A1
	$r = 2$	A1
Alt: Method γ	Circle meets given line	
	$(x - r)^2 + (y - 4)^2 = r^2$ with $4y = 3x$ substituted	M1
	$[\frac{25}{16}x^2 - (2r + 6)x + 16 = 0]$	
	Equal roots: $4r^2 + 24r - 64 = 0$	M1 A1
	Solving to give $r = 2$ ($r \neq -8$)	A1
Alt: Method δ	Using formula for distance of pt. from line	
	$AC = \frac{4(4) - 3(r)}{\sqrt{3^2 + 4^2}}$	M1 A1
	Equating to r and solving; $r = 2$	M1; A1
Alt: Method ε	$\tan 2\alpha = \frac{4}{3}$ and use double angle formula	M1
	$\Rightarrow \tan\alpha = \frac{1}{2}$; $\Rightarrow r = 2$	M1 A1 A1
Alt: Method ϕ	$\sin\theta = \frac{r}{\frac{16}{3} - r}$; $= \frac{3}{5}$ $\Rightarrow r = 2$	M1 A1

(ii)	$\theta + 2\alpha = \frac{1}{2}\pi$	M1
	$\arctan\left(\frac{3}{4}\right) + 2\arctan\left(\frac{1}{2}\right) = \frac{\pi}{2}$ AG (no errors, convincing)	A1 (8) cso
(b)		
Method α:	Tangent is perp. to given line; intercept = $(4 + r)\text{cosec}\,\theta$	M1 A1
	Complete method for q; e.g. 0 + 3(intercept) = q	M1
	$\Rightarrow q = 30$	A1 (4)
Method β	(Other variations)	
	Finding pt. on $4x + 3y = q$	M1 A1
	e.g where circle meets it $\left(\frac{18}{5}, \frac{26}{5}\right)$	
	where $4y = 3x$ meets it $\left(\frac{4q}{25}, \frac{3q}{25}\right)$ $\left(\frac{24}{5}, \frac{18}{5}\right)$	
	Complete method to find q, $q = 30$	M1 A1
		(12 marks)

Question Number	Scheme	Marks
5. (*a*)	Attempt at $\frac{\mathrm{d}y}{\mathrm{d}t}=\frac{f'(u)}{u}$;	M1
	$f'(u)=1+\frac{t}{\sqrt{(1+t^2)}}$ $\left[\ \frac{t+\sqrt{(1+t^2)}}{\sqrt{(1+t^2)}}\ \right]$	B1
	Completion: $\frac{\mathrm{d}y}{\mathrm{d}t}=\frac{1}{\sqrt{(1+t^2)}}$ AG	A1 (3) (cso)
(*b*) (i)	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{t}{tx+1}-\frac{1}{x}$ [t in numerator should be $t+x\frac{\mathrm{d}t}{\mathrm{d}x}$]	B1
(ii)	$\frac{\mathrm{d}x}{\mathrm{d}t}=-\frac{t}{(1+t^2)^{\frac{3}{2}}}$	M1 A1
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\mathrm{d}y}{\mathrm{d}t}.\frac{\mathrm{d}t}{\mathrm{d}x}=\frac{1}{\sqrt{(1+t^2)}}.-\frac{(1+t^2)^{\frac{3}{2}}}{t}$ $\left[=-\frac{(1+t^2)}{t}\right]$	M1 A1
	Correct complete argument	A1 (6)
(*c*)	$\ln\{-t+\sqrt{(1+t^2)}\}+\ln\{t+\sqrt{(1+t^2)}\}=\ln\{(1+t^2)-t^2\}$ or equiv	M1 M1
	$\ln\{-t+\sqrt{(1+t^2)}\}+\ln\{t+\sqrt{(1+t^2)}\}=0\Rightarrow$ result	A1 (3)
(*d*)	As $t\to -t$, $(x,y)\to(x,-y)$ [Accept that as enough, if fuller explanation not given]	B1
	 Asymptotic to y-axis, symmetric in x-axis	M1
	Correct curve, (1, 0) and no cusp	A1 (3)
		(15 marks)

Question Number	Scheme	Marks
6. (*a*)	 General shape	M1
	Marking 1 on y-axis	A1
	Totally correct (allow dotted vertical line but not full)	A1 (3)
(*b*)	$\int_2^p \text{f}(x)\text{d}x =$ area of shaded triangle or $\int_2^p (x-2)\text{d}x$	M1
	$= \frac{1}{2}(p-2)^2$	A1
	Setting equal to 0.18 to give $p = 2.6$	A1 (3)
(*c*)	Smallest root occurs where $\frac{1}{1+kx} = x$	M1
	Setting $x = ½ \Rightarrow k = 2$ (allow with no working)	A1 (2)
(*d*)	Sketch of $y = \text{g}(x)$ superimposed on $y = \text{f}(x)$ – see above	B1 (1)
(*e*)	Solution in $n < x < n + 1$: $\frac{1}{1+2x_n} = x_n - n$	M1 A1√ on *k*
	$\Rightarrow 2x^2 - (2n-1)x - (n+1) = 0$ c.s.o. * [M1 even if [*x*] for *n*]	M1 A1 (4)
(*f*)	Method using (e)	
	$x = \frac{2n-1+\sqrt{(2n-1)^2+8(n+1)}}{4} = \frac{2n-1+\sqrt{4n^2+4n+9}}{4}$	M1
	$\frac{2n-1+\sqrt{4n^2+4n+9}}{4} < n + 0.05$ [$\sqrt{4n^2+4n+9} < 2n + 1.2$]	A1
	$\Rightarrow 0.8n > 7.56 \Rightarrow n = 10$	M1 A1 (4)
Alt	*Alternative* : $\frac{1}{1+2x_n} = x_n - n \Rightarrow \frac{1}{1+2x_n} < 0.05$	M1 A1
	$\therefore 0.1x_n > 0.95 \quad \therefore x_n > 9.5$; $(n > 9.45)$ $n = 10$	M1; A1
	[Equality throughout lose final A1]	**(17 marks)**

Question Number	Scheme	Marks
7. (*a*)	$a^2 + b^2 = c^2$; $b^2 - a^2 = c^2 - b^2$	B1; B1
	Using the two equations to find b (or b^2) or c (or c^2) in terms of a	M1
	$c = \sqrt{3}\,a$, $b = \sqrt{2}\,a$	A1 (4)
(*b*)	$\cot A = \frac{b}{a} = \sqrt{2}$, $\cot B = \frac{a}{b} = \frac{1}{\sqrt{2}}$, $\cot C = 0$	M1A1
	Convincing conclusion that in *AP*, common difference = $\pm \frac{\sqrt{2}}{2}$ or equivalent	A1 (3)
(*c*)	 $h = q \sin P = p \sin Q$ or use area of Δ formula	M1
	$\Rightarrow \frac{p}{\sin P} = \frac{q}{\sin Q}$	A1
	Completion, e.g "by symmetry".	A1 (3)
(*d*) (i)	Using cosine rule: $\frac{\cos P}{p} + \frac{\cos R}{r} = \frac{(q^2 + r^2 - p^2)}{2pqr} + \frac{(p^2 + q^2 - r^2)}{2pqr}$ $\left(= \frac{q}{pr}\right)$	M1 A1
	and $\frac{2\cos Q}{q} = \frac{p^2 + r^2 - q^2}{pqr}$	M1
	Stating $p^2 + r^2 = 2q^2$ or equivalent	B1
	Using to show that they are equal	M1 A1 (cso) (6)
(ii)	Using two of the cosine formulae to give	
	$q^2 - p^2 = qr \cos P - pr \cos Q$ OR $r^2 - q^2 = pr \cos Q - pq \cos R$	M1 A1
	Forming other equation	M1
	Stating $p^2 + r^2 = 2q^2$ or equivalent	B1
	Using to give $2\,pr \cos Q = qr \cos P + pq \cos R$	M1
	Dividing through by prq to give result. *	A1
(e)	As $\frac{q}{\sin Q} = \frac{p}{\sin P} = \frac{r}{\sin R}$	
	$\therefore \frac{q}{\sin Q} \times \frac{2\cos Q}{q} = \frac{p}{\sin P} \times \frac{\cos P}{p} + \frac{r}{\sin R} \times \frac{\cos R}{r}$	M1 A1
	$\therefore 2\cot Q = \cot P + \cot R$ or equivalent	A1 (3)

Question Number	Scheme	Marks
Alt (*e*)	Using (d) and sine rule $2\cos Q \, . \dfrac{\sin R}{r\sin Q} = \cos P \, . \dfrac{\sin R}{r\sin P} + \dfrac{\cos R}{r}$ $\therefore 2\cot Q = \cot P + \cot Q$ or equivalent	 M1A1 A1 **(19 marks)**

STYLE, CLARITY and PRESENTATION MARKS

(a) **S marks**
For a novel or neat solution to any question, apply once per question in up to 3 questions.

S2 if solution is fully correct in principle, elegance and accuracy.

S1 if principle is sound but minor algebraic or numerical slip. S6 (S2 × 3)

(b) **T marks** T1
For a good and largely accurate attempt at the whole paper.